Marginal time-reversal and effective fluctuation relations

M. Polettini, B. Altaner, M. Esposito

University of Luxembourg

The response of thermodynamic systems slightly perturbed out of an equilibrium steady-state is described by two milestones of early nonequilibrium statistical mechanics: the reciprocal and the fluctuation-dissipation relations. At the turn of this century, the so-called “fluctuation theorems” extended the study of fluctuations far beyond equilibrium. All these results rely on the crucial assumption that the observer has complete information about the system: there is no hidden leakage to the environment, and every process is assigned its due thermodynamic cost. Such a precise control is practically unattainable and philosophically untenable, hence the following questions are compelling: Will an observer who has marginal information be able to perform an effective thermodynamic analysis? Given that such observer will only be able to establish local equilibrium amidst the whirling of hidden fluxes, by locally perturbing the stalling currents will he/she observe equilibriumlike fluctuations? Furthermore, the second law of thermodynamics is tightly intertwined with the concept of (global) time-reversal of the dynamics of a stochastic process, with the fluctuation relations for a complete set of currents quantifying the extent by which forward processes are more probable than backward ones. What if instead we restrict to a marginal subset of currents? We propose a notion of marginal time-reversal of a Markov jump process that allows to prove effective fluctuation relations for subsets of currents, and thus to establish thermodynamics (including the 2nd law and the fluctuation-dissipation relation) for an observer that only has partial information about a system. Marginally time-reversed generators have interesting mathematical properties related to concepts in graph theory and large deviation theory.

In particular we establish that: 1) While marginal currents do not obey a full-fledged fluctuation relation, there exist effective affinities such that an integral fluctuation relation holds; 2) Under reasonable assumptions on the parametrization of the rates, effective and “real” affinities only differ by a constant; 3) At stalling, i.e. where the marginal currents vanish, a symmetrized fluctuation-dissipation relation holds while reciprocity does not; 4) There exists a notion of marginal time-reversal that plays a role akin to that played by time-reversal for complete systems, which restores the fluctuation relation and reciprocity.

[l] Altaner, Polettini, Esposito, Phys. Rev. Lett. **117**, 180601 (2016).

[2] Polettini, Esposito, arXiv1703.05715.

[3] Polettini, Esposito, J. Stat. Mech. P10033 (2014).